Int. J. Heat Mass Transfer. Vol. 22, pp. 1159–1161
© Pergamon Press Ltd. 1979. Printed in Great Britain

0017-9310/79/0701-1159 $02.00/0

WIND AND SOLAR ENERGIES IN THE TORNADO TYPE WIND ENERGY SYSTEM

JWO-MIN CHEN
Power Mechanical Engineering Department, National Tsing-Hua University, Hsinchu, Taiwan 300, Republic of China

(*Received* 16 *August* 1978 *and in revised form* 18 *January* 1979)

NOMENCLATURE

b,	parameter defining the width of the volumetric suction force;
$\bar{B}$,	volumetric suction force;
B,	the strength of suction force;
c,	constant;
c_p,	specific heat;
E,	reduced pressure function;
F,	reduced radial velocity function;
G,	reduced tangential velocity function;
H,	reduced vertical velocity function;
k,	thermal conductivity;
p,	pressure;
q,	reduced value of t;
$\dot{Q}'''$,	volumetric line heat source;
$\dot{Q}'$,	line heat source per unit length;
r,	radial coordinate;
s,	reduced value of b;
t,	parameter defining the width of the line heat source;
T,	potential temperature;
u,	radial velocity;
U,	ru;
v,	tangential velocity;
V,	rv;
w,	vertical velocity;
W,	rw;
x,	similarity variable;
z,	vertical coordinate.

Greek symbols

ν,	kinematic viscosity;
α,	coefficient of thermal expansion;
θ,	reduced potential temperature function;
Γ,	circulation.

INTRODUCTION

THE CONVENTIONAL wind machine systems—such as the propeller and Darrieus types—are limited by some basic difficulties with harnessing wind energy efficiently. This occurs in situations where there is a low intensity of energy flux, where the wind has unpredictable fluctuations above the ground level, and where the long rotating blades will encounter large dynamic stresses and require increasing stiffness/weight requirements to maintain dynamic stability. There are two ways to increase power density: (1) increasing the local wind velocity; and (2) reducing the back pressure of the turbine. The tornado-type wind energy system which is reported by Yen [1] is one of those concentrators. This system uses a stationary tower with vertical vanes. The vanes directed the inflow wind so that it had a tangential component to supply circulation. The air exits from the opening top of tower. Then an internal vortex can be formed by this large hollow tower. Chen and Chao [2] establish a relationship between the vortex and the radius of the chimney which is placed over the top of the tower.

A number of analytical studies of the tornado-like vortex have been investigated by Long [3, 4], Kuo [5], and Chen and Watts [6]. Experimental studies have been carried out by Ying and Chang [7] who impart the vorticity by rotating a cylindrical hardware cloth screen, so that the updraft is produced by an exhaust fan at the opening of the top hood along the axis of the vortex. The model also serves as an analytical description of certain laboratory vortices created by electrical discharges such as those produced by Wilkins [8], Ryan and Vonnegut [9], and Watkins [10]. Watts and Chen [11] generate vortices around electrical discharges, the vorticity being provided by a rotating cage. They find that a very long and stable arc can be maintained for long periods when a drum is placed over the top of the centre of the cage to produce a chimney effect.

In this paper we report the results of an analytical model to the buoyancy-driven vortices about a line heat source which is from a solar collector in this tornado-type wind energy system (see Fig. 1).

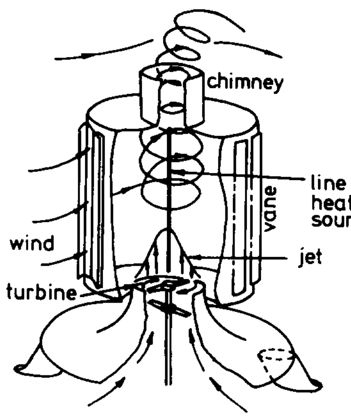

FIG. 1. Tornado-type wind energy system with heat source.

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the three-dimensional flow in a tornado-like vortex, with a core region of large vorticity, an outer region where the circulation, Γ_∞, is constant, and the vortex is driven by a line heat source created by a focusing solar collector. The work presented here is different from other research on buoyancy-driven vortices that currently appears in the literature. In this case flow is through a sink which is located at the top of the tower.

(a) *Governing equations*

The components of the momentum equation, energy equation, and continuity equation in cylindrical coordinates for steady, axisymmetric flow can be written as follows:

$$uu_r + wu_z - v^2/r = -p_r/\rho + \nu[u_{rr} + u_r/r - u/r^2 + u_{zz}] \quad (1)$$

$$u(rv)_r + w(rv)_z = \nu[(rv)_{rr} - (rv)_r/r + (rv)_{zz}] \quad (2)$$

$$uw_r + ww_z = -p_z/\rho + \nu[w_{rr} + w_r/r + w_{zz}] + \bar{B}/\rho + g\alpha T \quad (3)$$

$$uT_r + wT_z = K[T_{rr} + T_r/r + T_{zz}] + \dot{Q}'''/\rho c_p \quad (4)$$

$$(ru)_r + (rw)_z = 0. \quad (5)$$

In these expressions u, v, and w are the radial, tangential, and vertical velocities, p is the pressure difference between the local pressure and the pressure far from the centre and the volumetric suction force, $\bar{B}$, is defined by Chen and Chao [2] as follows:

$$\bar{B} = (B\varepsilon^c/\pi b)\exp(-r^2/b\sqrt{z}),$$

where B is a value which depends on the speed of wind which passes over the top of chimney and the height of chimney, and b is an adjustable constant. By choosing different values of b, a narrow or wide region of influence can be created while keeping B constant. By choosing b to be sufficiently large a fully opening top can be approximated. $\varepsilon = 1/Re$ (Reynolds number).

T is the potential temperature, α is the coefficient of volumetric thermal expansion, $K = k/\rho c_p$, c_p is the constant pressure specific heat, k is the thermal conductivity, and the heat source term, $\dot{Q}'''$, is defined by Chen and Watts [6] as follows:

$$\dot{Q}''' = (\dot{Q}'/\pi t\sqrt{z})\exp(-r^2/t\sqrt{z}),$$

where $\dot{Q}'$ is the rate at which heat is emitted from the solar collector per unit height, and t is an adjustable constant. By choosing various values of t a narrow or wide region of heat generating can be created while keeping $\dot{Q}'$ constant. By choosing t to be sufficiently small a line heat source can be approximated.

The dimensionless variables and constants are defined as follows:

$$L^3 = (\rho\Gamma_\infty^2/4\pi B)^2, \quad \bar{T} = Q'/\pi k, \quad Ch = g\alpha\bar{T}L/U_c,$$
$$U_c = \Gamma_\infty/2\pi L, \quad U' = ru/LU_c, \quad V' = rv/LU_c,$$
$$W' = rw/LU_c, \quad p' = p/\rho U_c^2, \quad T' = T/\bar{T},$$
$$r' = r/L, \quad z' = z/L, \quad \varepsilon = 2\pi\nu/\Gamma_\infty,$$
$$s = b/\varepsilon L^{3/2}, \quad q = t/\varepsilon L^{3/2}.$$

It is expected that in the core region the upward velocity is strong within a relatively narrow region where viscous effects are important. Using boundary layer arguments, it can easily be shown that when terms of order of magnitude ε are neglected compared with those of order unity in equations (1)–(5), the following equations result:

$$V^2 = r^3 p_r \quad (6)$$

$$UV_r + WV_z = rV_{rr} - V_r \quad (7)$$

$$UW_r + WW_z - UW/r = r[r(W/r)_r]_r + Chr^2T + (r^2/s)\exp(-r^2/s\sqrt{z}) \quad (8)$$

$$UT_r + WT_z = \sigma(rT_{rr} + T_r) + \sigma(r/q\sqrt{z})\exp(-r^2/q\sqrt{z}) \quad (9)$$

$$U_r + W_z = 0, \quad (10)$$

where

$$r = r'/\sqrt{\varepsilon}, \; z = z', \; p = p'/\sqrt{\varepsilon}, \; T = T', \; U = U'/\varepsilon,$$
$$V = V'/\varepsilon^{3/4}, \quad W = W'/\sqrt{\varepsilon}, \quad c = 1,$$

and

$$\sigma = 1/Pr \text{ (Prandtl number)}.$$

(b) *Boundary conditions*

The following physical boundary conditions must be satisfied:

1. The radial and tangential velocities are zero at the vortex centre;

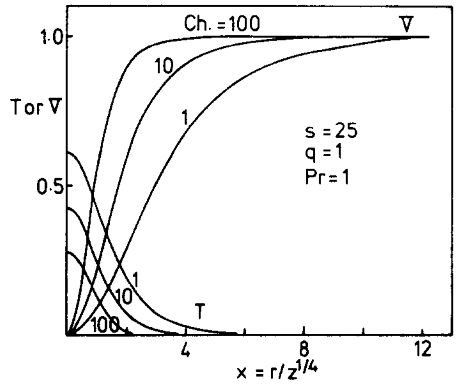

FIG. 2. Reduced potential temperature and reduced circulation in the core region.

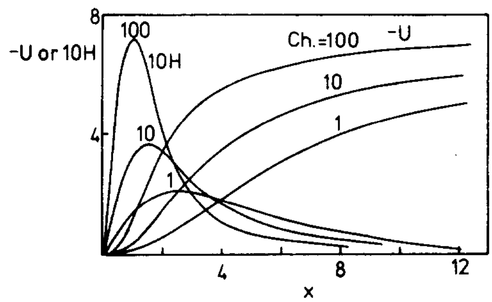

FIG. 3. Reduced vertical and radial velocity functions in the core region.

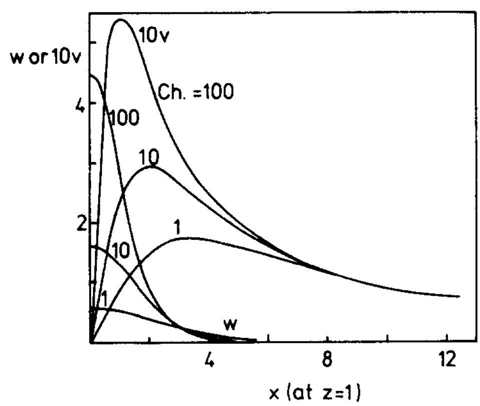

FIG. 4. Tangential and vertical velocities in the core region.

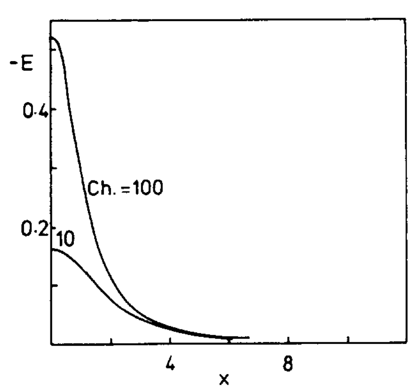

FIG. 5. Reduced pressure function in the core region.

2. The vertical velocity and the potential temperature are smooth at the vortex centre;

3. The circulation Γ about the vertical coordinate axis approaches a constant Γ_∞ far from the vertical axis; and

4. Each of the variables is bounded.

Mathematically, these boundary conditions are expressed as:

$$\begin{gathered} U(0,z) = V(0,z) = W(0,z) = T_r(0,z) = 0 \\ p(\infty,z) = W(\infty,z) = 0, \quad V(\infty,z) = 1 \\ T(\infty,z) = 0. \end{gathered} \tag{11}$$

(c) *Similarity equations*

The governing equations (6)–(10) can be transformed by the following changes of variables:

$$x = r/z^{1/4},\ U = F(x),\ V = G(x),\ W = 4z^{3/4}H(x),$$
$$p = E(x)/\sqrt{z},\ T = \theta(x). \tag{12}$$

Substituting (12) into equations (6)–(10), we obtain:

$$G^2 = x^3E' \tag{13}$$

$$FG' - xHG' = xG'' - G' \tag{14}$$

$$xFH' - x^2HH' + 3xH^2 - FH = H - xH' + x^2H'' + Chx^3\theta/4 + (x^3/4s)\exp(-x^2/s) \tag{15}$$

$$F\theta' - xH\theta' = \sigma(x\theta'' + \theta') + (\sigma x/q)\exp(-x^2/q) \tag{16}$$

$$F' - xH' + 3H = 0, \tag{17}$$

(a prime indicates differentiation with respect to x), with the boundary conditions:

$$\begin{gathered} F(0) = G(0) = H(0) = \theta'(0) = 0, \\ E(\infty) = H(\infty) = \theta(\infty) = 0,\ G(\infty) = 1. \end{gathered} \tag{18}$$

SOLUTIONS AND RESULTS

The set of ordinary differential equations (13)–(17) were reduced to eight first-order differential equations; direct integration using the Runge–Kutta method was used to obtain solutions to the equations. An iterative technique, the shooting method (written by Roberts and Shipman [12]), was used to choose new values of the missing initial conditions by adjoint equations. After the boundary conditions meet at the assumed edge, this edge is continually extended until convergence occurs.

Equations (12)–(15) were solved for several values of the parameter Ch that determines the strength of the heat source. Solutions were obtained for Ch = 0, 0.01, 0.1, 1, 10, and 100 with $s = 25$, $q = 1$, and $Pr = 1$.

The results are shown in Figs. 2–5. The dimensionless circulation and potential temperature are plotted against the similarity variable x in Fig. 2. Figure 3 shows the dimensionless variables U and H.

The dimensionless tangential velocity and the vertical velocity are shown in Fig. 4. Figure 5 shows the dimensionless pressure distribution. It is found that the dimensionless potential temperature is decreased as Ch increases, but the dimensional potential temperature will increase as Ch increases. In the table, the maximum pressure drop at the vortex centre axis is increased as Ch increases.

Table 1. The minimum pressure drop with various Ch values

Ch	0	0.01	0.1	1.0	10.0	100
$(-p)\times 10^2$	1.586	1.633	2.407	5.547	16.246	52.832

CONCLUSIONS

The effect of a line heat source on a tornado-type wind energy generator has been studied. Using the similarity technique, a set of ordinary differential equations has been derived. Solutions have been obtained by a numerical method for several values of the parameter Ch. It is found that the heat from the solar collector affects the tangential velocity and pressure distribution of the vortex. The maximum pressure drop at the vortex centre axis will increase as the strength of the line heat source increases.

REFERENCES

1. J. T. Yen, Tornado-type wind energy system, Tenth Intersociety Energy Conversion Engineering Conference, IEEE Catalog No. 75CHO 983-7 TAB. (1975).
2. J. M. Chen and H. H. Chao, Numerical simulation of the tornado-type wind energy system with a chimney effect, Annual Report, National Tsing-Hua Univ. PME-01 (1977).
3. R. R. Long, Vortex motion in a viscous fluid, *J. Meteorol.* **15**, 108 (1958).
4. R. R. Long, A vortex in an infinite viscous fluid, *J. Fluid Mech.* **11**, 611 (1961).
5. H. L. Kuo, On the dynamics of convective atmospheric vortices, *J. atmos. Sci.* **23**, 25 (1966).
6. J. M. Chen and R. G. Watts, A steady vortex driven by a line heat source, *Int. J. Heat Mass Transfer* **00**, 000 (1979).
7. S. J. Ying and C. C. Chang, Experimental model study of tornado-like vortex dynamics, *J. atmos. Sci.* **27**, 1 (1970).
8. E. Wilkins, The role of electrical phenomena associated with tornadoes, *J. geophys. Res.* **69**, 2435 (1964).
9. R. T. Ryan and B. Vonnegut, Miniature whirlwinds produced in the laboratory by high voltage electrical discharges, *Science* **168**, 1349 (1970).
10. D. Watkins, The properties of a vortex-stabilized discharge in the current range of 3 to 300 amperes and an estimation of the ability of electrical discharge to power tornadoes, Ph.D. Dissertation, SUNY-Albany (1975).
11. R. G. Watts and J. M. Chen, Theory of prester tornadoes, Annual Report, NSF Grant GK-35993 (1974).
12. S. M. Roberts and J. S. Shipman, *Two-point Boundary Value Problems: Shooting Methods*, p. 17. American Elsevier, New York (1972).